MCS 282 Exam II, Spring Semester 2010
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1) (35 points)
Let us consider a clamped cubic spline interpolating the data points (x_i, y_i), $i = 1, 2, 3$. Denote the cubic polynomials S_1 and S_2 for each of the subintervals $[x_1, x_2]$ and $[x_2, x_3]$. Define

$$S_1(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3,$$
$$S_2(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3.$$

(i) Write down the conditions to construct the clamped cubic spline.

(ii) Construct the system in a matrix form to solve the coefficients $a_0, a_1, \ldots, b_0, \ldots, b_3$.

(iii) If we want to use natural cubic spline, what will change? Explain the difference.

2) (40 points)
The divided difference table corresponds to the data is given as below. (Note: Use 7-digit rounding method.)

a) Find the missing entries in the table. Show all your calculations.

x_i	$f[x_i]$	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$	$f[x_{i-3}, x_{i-2}, x_{i-1}, x_i]$	$f[x_{i-4}, x_{i-3}, x_{i-2}, x_{i-1}, x_i]$
0.0	1.0000000				
1.0	0.5403023	?			
2.0	?	-0.9564491	?		
3.0	-0.9899925	-0.5738457	0.1913018	?	
4.0	-0.6536436	?	0.4550973	?	?

b) We want to approximate $f(0.05)$, $f(3.8)$ and $f(2.2)$. Approximate these points in a best way.

3) (25 points) Use the formulas given in section 4.1 to determine, as accurately as possible, approximations for each missing entry in the following table:

x	$f(x)$	$f'(x)$	$f''(x)$
-3.0	9.367879	?	
-2.8	8.233241	-5.468933	
-2.6	7.180350	?	?
-2.4	6.209329	-4.650223	
-2.2	5.320305	-4.239911	