MCS 282 Final Examination, Spring Semester 2010
Department of Mathematics & Computer Science, Çankaya University

1) (25 points)
In calculus, we learn that $e = \lim_{h \to 0}(1+h)^{1/h}$.

(i) Compute approximations to e using the formula $N(h) = (1+h)^{1/h}$ for $h = 0.04, 0.02$ and 0.01.

(ii) Use extrapolation on the approximations, assuming the constants K_1, K_2 ...exist with $e = N(h) + K_1 h + K_2 h^2 + K_3 h^3 + \ldots$ to produce an $O(h^3)$ approximation to e where $h = 0.04$.

(iii) Note that, e also equals to as follows:

$$e = \lim_{h \to 0}\left(\frac{2+h}{2-h}\right)^{1/h}.$$

Let $N(h) = (\frac{2+h}{2-h})^{1/h}$. Show that $N(h) = N(-h)$.

(*Note: In your calculations use 5-digit rounding*)

2) (20 points)
Consider the integration of the function $f(x) = 1 + xe^x - e^{2x}$ over $[a, b] = [0, 1]$. Use exactly 5 function evaluations to compare the results from Composite Simpson's rule, Composite Midpoint rule and Boole's rule. Calculate the exact value and find the absolute errors. (*In your calculations use 4-digit rounding*)

3) (10 points)
Fill in the missing entries in the table. Write down all your explanations explicitly.
Root finding:

Method	Iteration equation	Assumptions
Fixed Point	? (1)	? (2)
? (3)	$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$	? (4)
Secant	? (5)	$f'(p_{n-1}) \approx \frac{f(p_{n-1})-f(p_{n-2})}{p_{n-1}-p_{n-2}}$

Interpolation: Assume we have $n+1$ distinct nodes, $x_0, x_1, \ldots, x_n$ in $[a, b]$. Suppose that $f \in C^{(n+1)}[a, b]$. We want to approximate f by a polynomial of degree n, denoted by $P(x)$, by using the following methods in the table. Fill in the missing entries.

Method	Number of Conditions	Error Term
Lagrange	? (1)	$\frac{f^{(n+1)}(c(x))}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n)$
Divided Difference	$n+1$	? (2)
Cubic Spline	? (3)	-
? (4)	? (5)	$\frac{f^{(2n+2)}(c(x))}{(2n+2)!}(x-x_0)^2(x-x_1)^2\cdots(x-x_n)^2$

4) (15 points) Find the constants c_0, c_1, x_1 so that the quadrature formula

$$\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision.

5) (15 points) A natural cubic spline S is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + B(x-1) - D(x-1)^3 & \text{if } 1 \le x < 2 \\ S_1(x) = 1 + b(x-2) - \frac{3}{4}(x-2)^2 + d(x-2)^3 & \text{if } 2 \le x \le 3 \end{cases}$$

If S interpolates the data $(1, 1)$, $(2, 1)$ and $(3, 0)$, find the constants B, D, b, d.

6) (15 points) Show that

$$f[x_0, x_1, \ldots, x_n, x] = \frac{f^{(n+1)}(c(x))}{(n+1)!},$$

for some $c(x)$. [Hint: Consider the interpolation polynomial of degree $n+1$ on $x_0, x_1, \ldots, x_n, x$.]